$\boxed{8.1}$

$$\int_0^1 \frac{x+3}{\sqrt{4-x^2}}\,dx$$

Let $u = 4 - x^2$

$\frac{du}{dx} = -2x$

$dx = -\frac{1}{2}\,du$

$$= \int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx + 3\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$$

$a = 2$

$$= -\frac{1}{2}\int_4^3 \frac{1}{\sqrt{u}}\,du + 3\int_0^1 \frac{1}{\sqrt{a^2-x^2}}\,dx$$

$$= -\frac{1}{2}\int_4^3 u^{-1/2}\,du + 3\left[\arcsin\left(\frac{x}{2}\right)\right]_0^1$$

$$= -\frac{1}{2}\left[2u^{\frac{1}{2}}\right]_4^3 + 3\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1$$

$$= -\left[\sqrt{3} - \sqrt{4}\right] + 3\left[\sin^{-1}\left(\frac{1}{2}\right) - \arcsin(0)\right]$$

$$= 2 - \sqrt{3} + \frac{\pi}{2}$$

$$\simeq 1.83874552$$

$$\int \frac{4x}{x^2+9}\,dx$$

Let $u = x^2+9$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{1}{2x}\,du$$

$$= \int \frac{4x}{u}\left(\frac{1}{2x}\right)du$$

$$= 2\int \frac{1}{u}\,du$$

$$= 2\ln|u| + C$$

$$= 2\ln|x^2+9| + C$$

To check using calculator, enter the final answer into ($Y=$) Y_1, and turn Y_1 graph off.
In Y_2, enter nDeriv(Y_1, X, X)
In Y_3, enter the original integrand

$\boxed{8.1}$ # 45

$$\int \frac{\tan\left(\frac{2}{t}\right)}{t^2}\,dt$$

Let $u = \frac{2}{t}$

$\frac{du}{dt} = -\frac{2}{t^2}$

$dt = -\frac{t^2}{2}\,du$

$$= \int \frac{\tan(u)}{t^2}\left(-\frac{t^2}{2}\right)du$$

$$= -\frac{1}{2}\int \tan(u)\,du$$

= [illegible]

= [illegible]

$$= \frac{1}{2}\ln\left|\cos\left(\frac{2}{t}\right)\right| + C$$

$\boxed{8.2}$ # 35

$$\int e^{2x} \sin(x)\,dx$$

Let

$u = e^{2x}$	$dv = \sin x\,d$
$\frac{du}{dx} = 2e^{2x}$	$v = -\cos(x$
$du = 2e^{2x}dx$	

$$= \int u\,dv$$

$$= uv - \int v\,du$$

$$= \overset{u}{(e^{2x})}\overset{v}{(-\cos(x))} - \int \overset{v}{(-\cos(x))}\overset{du}{(2e^{2x}dx)}$$

$$= -e^{2x}\cos(x) + 2\int \cos(x)\, e^{2x}\,dx$$

Let

$u = e^{2x}$	$dv = \cos x$
$\frac{du}{dx} = 2e^{2x}$	$v = \sin x$
$du = 2e^{2x}dx$	

$$= -e^{2x}\cos(x) + 2\left[\overset{u}{(e^{2x})}\overset{v}{(\sin x)} - 2\int \overset{v}{(\sin x)}\overset{du}{(e^{2x}dx)}\right]$$

$$= -e^{2x}\cos(x) + 2e^{2x}\sin x - 4\int \sin x\, e^{2x}\,dx$$

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin x + C$$

$$\int e^{2x}\sin x\,dx = \frac{-e^{2x}\cos(x) + 2e^{2x}\sin x}{5} + C$$

$\boxed{8.3}$ #31

$$\int \tan^5\left(\frac{x}{2}\right) dx$$

$$= \int \tan^3\left(\frac{x}{2}\right) \tan^2\left(\frac{x}{2}\right) dx$$

$$= \int \tan^3\left(\frac{x}{2}\right)\left(\sec^2\left(\frac{x}{2}\right) - 1\right) dx$$

$$= \int \tan^3\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx - \int \tan^3\left(\frac{x}{2}\right) dx$$

$$= 2\int u^3\, du - \int \tan^3\left(\frac{x}{2}\right) dx$$

$$= \frac{1}{2} u^4 - \int \tan\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx - \int \tan\left(\frac{x}{2}\right) dx$$

$$= \frac{1}{2} u^4 - \left[2\int v\, dv + 2 \ln\left|\cos\left(\frac{x}{2}\right)\right|\right]$$

$$= \frac{1}{2} u^4 - v^2 - 2 \ln\left|\cos\left(\frac{x}{2}\right)\right| + C$$

$$= \frac{1}{2} \tan^4\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right) - 2 \ln\left|\cos\left(\frac{x}{2}\right)\right| + C$$

Let
$u = \tan\left(\frac{x}{2}\right)$
$\frac{du}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$
$dx = \frac{2}{\sec^2\left(\frac{x}{2}\right)}$

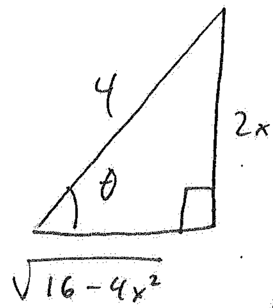
Let
$v = \tan\left(\frac{x}{2}\right)$
$\frac{dv}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$
$dx = \frac{2}{\sec^2\left(\frac{x}{2}\right)} dv$

$\boxed{8.4}$ #25

$$\int \frac{(16-4x^2)^{\frac{1}{2}}}{1}\,dx$$

Key: $\sqrt{a^2-u^2}$

$a^2 = 16$	$u^2 = 4x^2$
$a = 4$	$u = 2x$



$$\frac{2x}{4} = \sin\theta$$

$$2x = 4\sin\theta$$

$$x = 2\sin\theta$$

$$\frac{dx}{d\theta} = 2\cos(\theta)$$

$$dx = 2\cos(\theta)d\theta$$

$$\cos\theta = \frac{\sqrt{16-4x^2}}{4}$$

$$4\cos\theta = \sqrt{16-4x^2}$$

$$= \int (4\cos\theta)(2\cos(\theta)\,d\theta)$$

$$= 8\int \cos^2(\theta)\,d\theta$$

$$= 8\int \frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= 4\int (1+\cos(2\theta))\,d\theta$$

$$= 4\int d\theta + 4\int \cos(2\theta)\,d\theta$$

$$= 4\theta + \frac{4}{2}\sin(2\theta) + C$$

$$= 4\arcsin\left(\frac{x}{2}\right) + 2\left(2\sin\theta\cos\theta\right) + C \leftarrow \text{Double angle}$$

$$= 4\arcsin\left(\frac{x}{2}\right) + 4\left(\frac{x}{2}\right)\left(\frac{\sqrt{16-4x^2}}{4}\right) + C$$

$$= 4\arcsin\left(\frac{x}{2}\right) + \frac{x\sqrt{16-4x^2}}{2} + C$$

TEST REVIEW

D. B. (7)
REVIEW

8.5

2/24/12

#15) $\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} dx$

$2x \quad \frac{x+5}{x^2-2x-8}$

$x^2 - 2x - 8 \overline{\big) 2x^3 - 4x^2 - 15x + 5}$
$- \underline{2x^3 - 4x^2 - 16x}$
$x + 5$

$= \int \left[2x + \frac{x+5}{x^2-2x-8}\right] dx$

$= 2\int x\,dx + \int \frac{x+5}{x^2-2x-8} dx$

$= \cancel{2}\left[\frac{x^2}{\cancel{2}}\right] + \int \left[\frac{\frac{3}{2}}{x-4} + \frac{-\frac{1}{2}}{x+2}\right] dx$

$\frac{x+5}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$

$x + 5 = A(x+2) + B(x-4)$

let $x = 4$ $\quad$ let $x = -2$

$A(6) + B(0) = 9 \quad A(0) + B(-6) = 3$

$A\left(\frac{3}{2}\right) \quad -6B = 3$

$B = -\frac{1}{2}$

$= x^2 + \frac{3}{2}\int \frac{1}{x-4} dx + -\frac{1}{2}\int \frac{1}{x+2} dx$

$= x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C$

8.7 #33

$$\lim_{x\to\infty} \frac{\cos(x)}{x} = 0$$

$$-1 \le \cos(x) \le 1$$

$$-\frac{1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$$

$$\lim_{x\to\infty} -\frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

By THE SQUEEZE THEOREM

$$0 \le \lim_{x\to\infty} \frac{\cos x}{x} \le 0$$

$$\text{So} \lim_{x\to\infty} \frac{\cos(x)}{x} = 0$$

#43) $$\lim_{x\to\infty} \frac{\int_1^x \ln(e^{4t-1})\,dt}{x}$$

$$= \lim_{x\to\infty} \frac{\int_1^x (4t-1)\,dt}{x}$$

$$= \lim_{x\to\infty} \frac{\frac{d}{dx}\left[\int_1^x (4t-1)\,dt\right]}{\frac{d}{dx}\,x}$$

$$= \lim_{x\to\infty} \frac{4x-1}{1} = \infty$$

9

8.7

$$\lim_{x \to 0^+} (1+x)^{1/x} = 1^\infty$$

LET $y = \lim_{x \to 0^+} (1+x)^{1/x}$

$$\ln(y) = \ln\left[\lim_{x \to 0^+} (1+x)^{1/x}\right]$$

$$\ln(y) = \lim_{x \to 0^+} \ln\left[(1+x)^{1/x}\right]$$

$$\ln(y) = \lim_{x \to 0^+} \left[\frac{1}{x} \cdot \ln(1+x)\right]$$

$$\ln(y) = \lim_{x \to 0^+} \left[\frac{\ln(1+x)}{x}\right]$$

$$= \frac{\ln(1)}{0} = \frac{0}{0}$$ STOP LHOPITAL

$$\ln(y) = \lim_{x \to 0^+} \frac{\frac{d}{dx}\left[\ln(1+x)\right]}{\frac{d}{dx}[x]}$$

$$\ln(y) = \lim_{x \to 0^+} \frac{\frac{1}{1+x}}{1} = \ln(y) = \frac{1}{1}$$

$$\ln(y) = 1$$

$$e^{\ln(y)} = e^1$$

$$y = e$$

$$\boxed{\lim_{x \to 0^+} (1+x)^{1/x} = e}$$

10

let $u = -4x$
$du = -4dx$
$-\frac{du}{4} = dx$

$x = b$
$u = -4b$

$x = 0$
$u = -4(0)$
$u = 0$

8.8 #13) $\int_0^\infty e^{-4x}\,dx$

$= \lim_{b \to \infty} \int_0^b e^{-4x}\,dx$

$= -\frac{1}{4} \lim_{b \to \infty} \int_0^{-4b} e^u\,du$

$= -\frac{1}{4} \lim_{b \to \infty} \left[e^u\right]_0^{-4b}$

$= -\frac{1}{4} \lim_{b \to \infty} \left[e^{-4b} - e^0\right]$

$= -\frac{1}{4} \cdot \lim_{b \to \infty} \left[\frac{1}{e^{4b}} - 1\right]$

$= -\frac{1}{4} \cdot (0 - 1)$

$= -\frac{1}{4}(-1)$

$= \frac{1}{4}$

TEST REVIEW

$u = 8 - x$

$x = 0$

$u = 8$

$x = b$

$u = 8 - b$

$\frac{du}{dx} = -1$

$\frac{du}{-1} = dx$

#34 $\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx$

$= \lim_{b \to 8^-} \int_0^b \frac{1}{\sqrt[3]{8-x}}\,dx$

$= \lim_{b \to 8} -\frac{3}{2}\left[(8-b)^{2/3} - (8-0)^{2/3}\right]$

$= \lim_{b \to 8} -\frac{3}{2}(8-b)^{2/3} + 6 \qquad = \int_{u=8-b}^{u=0} u^{2/3}(-du)$

$= -\frac{3}{2}(8-8)^{2/3} + 6$

$= 6$